Last Diminisher Improves Balance in Settlers of Catan

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Abstract
The Settlers of Catan Xbox 360 and board games exhibit a common flaw among turn-based games: the first player has a huge advantage. We show how to adapt an economics theory for fair division called Last Diminisher into a practical mechanic that improves fairness by 3x, and thus gameplay balance, in Settlers and related games.

Background
In the first round of Settlers of Catan, each player claims two vertices on a graph representing an island. Each face of the graph is labeled with one of five types of resource and the rate at which adjacent vertices produce that resource. A player’s income rate is thus the sum of the rates on faces adjacent to his or her claimed vertices.

The true value of a vertex pair is a function $V$ of strategic board position, the combination of resources produced, and the income rate. This function is too complicated to ask players to compute explicitly when valuing positions, and as in any negotiation, it is in each player’s interest to misrepresent their heuristic evaluation of the function to gain an unfair advantage.

For an N-player game, let the collective value of the best 2N vertices be $T$. We desire a vertex allocation that is fair (all players receive equal value) and efficient (that value is $T/N$). Because positions are discrete, it is desirable to have the closest approximation to a fair and efficient solution. Furthermore, that solution must arise despite all players acting in their own best interests. In game theory terms, we are trying to choose rules that align the equitable, efficient, and Nash equilibria when players are rational maximizers. We can’t compute the values or $T$ without a full game-tree search because $V$ is complicated, but our solution only depends on rational maximizers ranking values, which they can do even if they cannot determine their absolute value.

Under the original rules, Settlers is not fair because the first player’s optimal strategy is to claim the vertex with the highest income (holding resource distribution and strategic position fixed, a fair allocation should have equal income.) In the worst case for $N=4$, player 1 has a 23% income advantage over player 4 (13 vs. 10) from the first vertex under optimal play, and that may be higher if we take resource distribution into account. The rules specify that the turn order when claiming the second vertex is the reverse of the first, but in practice this fails to balance the first player’s advantage because the second claim usually ensures a resource mixture and does not affect the quantity of the income.

Allocating the best 2N vertices fairly reduces to the problem of dividing a good (e.g., a cake) into N equal pieces, for which there exist many theoretical solutions in economics literature. However, previous algorithms [1] (e.g., Selfridge-Conway Discrete, Banach-Knaster Last-Diminisher, Austin’s Moving Knife) are impractical because they require either a continuous good or an infinite number of operations.

Our Last Diminisher Modification
Our modification adapts Banach and Knaster’s fair division method to the discrete space and practical constraints of Settlers. Players repeat the following steps until all players have been assigned two vertices:

1. The first player without a pair of vertices proposes to claim a specific pair whose incomes sum to $X$.
2. Each player may either pass or become the new claimant and move the vertices such that the new net income is $X' < X$.
3. When play passes all of the way around the table to the current claimant, that player receives the two vertices and is removed from the claim process.

The final player chooses any pair with income no more than any other player.

Analysis
Without loss of generality, consider the situation of player $P$’s choice. Let $S$ be the set of all currently possible pair valuations; $P$ is choosing $x_0 \in S$. The fair solution is for everyone to select $x=T/N$ on their own turn. There are three reasons that cannot occur: 1) it is a discrete problem, so $x=T/N$ is rarely available; 2) players are able to identify $x=\max(S)$, but not necessarily $x=T/N$ (they are rational maximizers, not rational fair sharers); and 3) players maximize their own outcome, not fairness. Our modification provably approaches the fair and efficient outcome despite players choosing selfishly.

Let $x^* \in S$ s.t. $x^* \geq T/N$ and there does not exist an $y \in S$ s.t. $T/N \leq y < x^*$. Thus $x^*$ is the best possible choice for the current player; it is better than a fair share, but only so slightly that no-one would claim a diminished variation. $P$ has at most four choices, shown in the game tree below:

If $P$ is able to judge $x^*$ accurately, he will select (c), which is the optimal minimax choice. If $P$ is irrationally optimistic or unable to judge $x^*$, he will choose (d), leaving the next player the same set of choices. This results in $P$ receiving at best a fair share or, more likely, a less than fair share that is left over when all other players have chosen (when $T/N$ is not available). Thus $P$ will choose (c), meaning that all players choose as fairly and efficiently as possible. We limit the first player’s advantage to discrete roundoff of $T/N$ (at most 1/12, about 8%), thus making the game about “three times more fair” than the original.