Last Diminisher Improves Balance in Settlers of Catan

Problem
The *Settlers of Catan* Xbox 360 (fig. 1) and board games exhibit a common flaw among turn-based games: the first player can gain a huge advantage by claiming the most valuable real-estate. *Settlers* begins with players claiming vertices on a graph that represents an island. Fig. 2 shows that player 1 gains a 30% income advantage over player 4 after the first round with optimal play; since they tend to select their next vertex to ensure resource distribution (not shown), player 4 cannot recover.

The game needs a new mechanic that does not change the character of play yet provides a fair allocation of real-estate for the first round.

Contributions
1. Guarantees a fair allocation when one exists
2. Minimizes the first player’s advantage in the worst case
   a. 3x better than *Settlers* rules
3. Generalizes an existing Economics method
   a. Applies to auctions, government spending, negotiation
   b. Extends Last Diminisher[1] to discrete spaces
   c. Decouples diminishing quantity from payoff

Figure 1. Xbox 360 video game *Settlers of Catan.*

Figure 2. In unmodified *Settlers*, players choose these positions, giving player 1 a 30% income advantage over player 4.

Figure 3. Under optimal play, Last Diminisher produces a fair allocation on the same board (verified by game tree search*).

Our “Last Diminisher” Modification
In our modification, *N* players repeat the following steps until *N*-1 players have been assigned two vertices:

1. The first player without a pair of vertices proposes to claim a specific pair whose incomes sum to *X*.
2. Each player may either pass or diminish the claim to any two vertices with net income to *X’* < *X*, replacing the previous claim.
3. When play passes all of the way around the table back to the last diminisher, that player receives the two vertices and is removed from the claiming process.

The last player chooses any pair with income *X* ≤ max(*X*1, ..., *X*<sub>*N*−1</sub>).

Results
Our modification ensures the most efficient fair outcome using rules that are practical for use in the real game. For simplicity, we only show the case of choosing a single vertex based on income in figs 2-3 to demonstrate the problem and solution. For the full case of two vertices and payoffs based on position, income, and resource distribution, we have preliminary positive results from a game theory proof, exhaustive search of the game tree, and playtests with real players. The following is an outline of a weaker proof for the case where the payoffs are income.

Let *S* be the set of all currently possible pair valuations and *z* be the fairest value for everyone to choose (which may not actually be in *S*). Player *P* is choosing *x* ∈ *S*. Let *x* ∈ *S* s.t. *x* ∈ *z* and there does not exist a *y* ∈ *S* s.t. *y* < *x* ∈ *S*. Thus *x* ∈ *S* is the best possible choice for the current player; it is better than a fair share, but only so slightly that no-one would claim a diminished variation. *P* has at most four choices, shown in the game tree below. Inductive proof shows *P* must choose (c) or risk being forced to accept *x* < *z* later; he can never retain a value *x* > *x*<sup>*</sup> without another player diminishing it.

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**References**